Microwaves and Mathematics

Mathematics and mathematicians play an essential role in the development of science and engineering. This is particularly conspicuous in the case of electromagnetic theory and microwave engineering. In the preface to his famous treatise, Maxwell refers to Gauss as the man who " . . . brought his powerful intellect to bear on the theory of magnetism, and on the methods of observing it, and [who] not only added greatly to our knowledge of the theory of attractions, but reconstructed the whole of magnetic science as regards the instruments used, the methods of observation, and the calculation of the results. . . . " Maxwell also adds: "The great success which these eminent men [Gauss, Weber, Riemann, J. & C. Neumann, Lorentz, etc.] attained in the application of mathematics to electrical phenomena, gives, as is natural, additional weight to their theoretical speculations. . . . " But a note of dissatisfaction is sounded in "There is also a considerable mass of mathematical memoirs which are of great importance in electrical science, but . . . they are for the most part beyond the comprehension of any but professed mathematicians."

As a matter of fact, the influence of mathematics on the development of science transcends direct application of mathematical methods either in establishing precise relations between observable physical quantities or in solving specific problems. Even more important is the role of mathematics in the creation of "physical" concepts and suitable models to aid our understanding and mastery of natural phenomena. First comes a recognition of essential similarities in apparently different physical phenomena; that is, a discovery of the analogy between them. Then follow abstractions and generalizations. As mathematical concepts grow and age, they gather flesh about them and gradually become "physical" concepts. This happened to the energy concept and the impedance concept. And somewhere creative imagination enters the picture.

Examples are plentiful. We have Lord Kelvin who formulated his theory of electrical discharge of a Leyden jar by drawing a parallel between electrical and mechanical phenomena. We have Faraday who created a model of an electromagnetic field which is essentially geometric even though he endowed his "lines of force" and "tubes of force" with some mechanical properties. And it takes a mathematician to make bold use of analogies and pictures without being ashamed of it. How many mathematical equations do we find in *Maxwell's Theory and Hertzian Oscillations*, a book of over one hundred pages by Henri Poincaré? None.

Microwave theory and engineering are particularly indebted to mathematics in its various forms: to Faraday's basic mathematical model of an electromagnetic field, to Maxwell's imaginative concept of displacement current and to his translation of Faraday's geometric language into analytic, thus enabling Hertz to calculate the field around a dipole and Lord Rayleigh to predict that electromagnetic waves of sufficiently high frequency can travel inside cylindrical tubes. While some mathematical predictions might have been anticipated by experimental discoveries, there are instances in which this would have been extremely unlikely. To establish experimentally that the attenuation of a circular electric wave decreases with increasing frequency would have required a tremendous amount of effort and ingenuity and expense to keep the wave "clean." Would anyone have been willing to make the investment without some idea of the possible outcome?

Abstract model building in the microwave theory is not over. About a century ago, Lord Kelvin formulated his telegraphist's equations for cables, or electric transmission lines as we now call them. Then, in more recent times, the concept of "normal transmission mode" was evolved which enabled us to think of coupled transmission lines as a certain number of uncoupled transmission lines. More recently still, the concept of waveguide mode was further generalized so that we can think of each normal mode as the result of superposition of coupled modes belonging to a rather arbitrarily chosen set. In this way it was possible to obtain from Maxwell's equations generalized telegraphist's equations for quite general types of waveguides. By this generalization, the gap between Kelvin's theory of transmission lines, based on the concept of distributed circuit parameters, and Maxwell's field theory was closed.

The future? Mathematics should continue to exert great influence on microwave theory and engineering. There certainly will be continued problem solving. And, who knows, some ingenious mind may create a model which will throw a new light on diffraction phenomena, or present us with a particularly satisfying way of thinking about propagation in large waveguides.

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